Structural analysis of the main apse vault of St. George of Greeks Cathedral built c.1390 at Famagusta, Cyprus

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Abstract

Foundation date of the Cathedral Church of St. George of the Greeks[1] can be dated with a high degree of probability to the end of the XIV century. This magnificent building was built on the edge of the Greek quarter, which occupied almost the whole of the southern end of the city of Famagusta.

The history says that this building was abandoned after 1571, as it had suffered severely from the fire of the battery established by the Turks on the rock to the south-east of the harbour and the marks of cannon-balls can be seen on the walls of the apse.

The building was built so strong that it could stand any kind of battering and to earthquake to a certain extend. In detail survey was done to find out the building technique and materials used, including the quarry where the stones were cut, the chemical composition and the strength of the stones and mortar used in between. A very sophisticated structural analysis[2] was carried to find out the strength of the main vault, its behavior and reactions to external forces, especially to earthquake and cannon balls.

Location

Cathedral Church of St. George of the Greeks is situated in the south east part of the town of Famagusta on the east cost of the Cyprus island, located at the eastern part of Mediterranean sea. It's exact position is $35^0$07.2' N, $33^0$56.7' E.

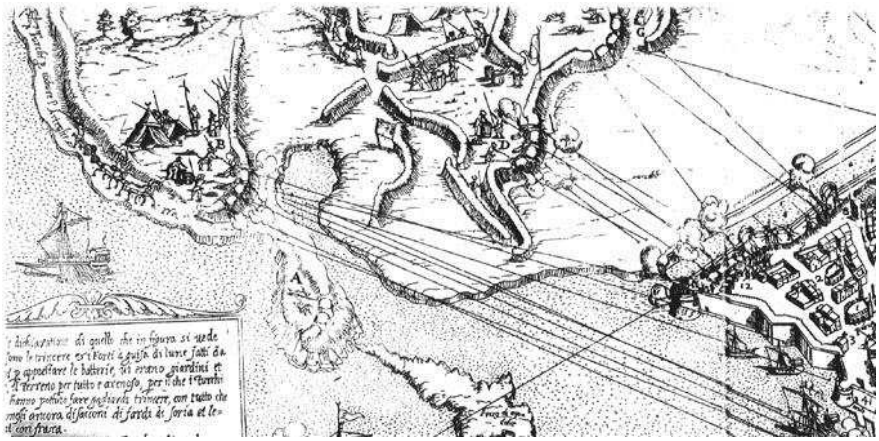

Figure 1: Part of the Stephan Gibellino's gravure detailing the siege of Famagusta. Ottoman cannons (Marked B), Cathedral Church of St. George of the Greeks (Marked 2)

The Ottoman cannonballs battered the church were fired from;

a) The cannons located on the rock to the south east of the harbour.[3]
b) The galleys outside the port.[3]

Mainly the cannonballs fired from the cannons positioned on the rock to the south east of the harbour hit and damaged the Cathedral.

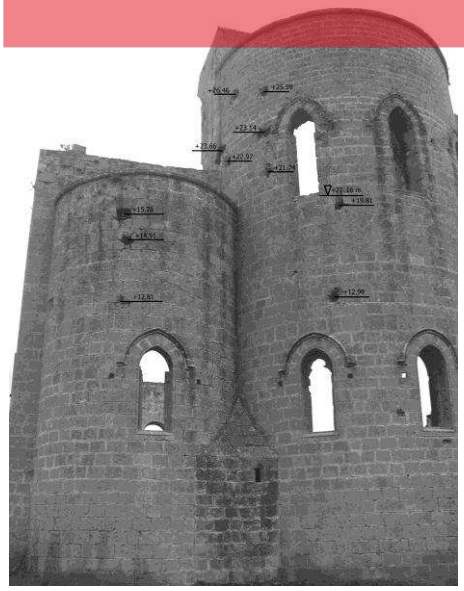

Figure 2: Marks and levels of cannon-balls on the walls of the apse. (Rear façade)

Wall details of the Cathedral

Main building outer walls

Thickness : 123-125 cm.
Outer layer : 37 cm. thick yellow sand stone, sized 25 (w) x 50 (l) x 35 (h)cm.
Middle layer : 50 cm. (smaller size yellow sand stones embedded in plaster.)
Inner layer : 37 cm. thick yellow sand stone.

Apse walls

Thickness : 72 cm.
Outer layer : 37 cm.
Middle layer : 10 cm. (smaller size yellow sand stones embedded in a plaster)
Inner layer : 25 cm.

Yellow sand stone specifications

Density : 1.86 kg/m^3 [4]
Compressive strength : 62.50 kg/cm^2 [4]
Modulus of rupture : 14 kg/cm^2 [4]

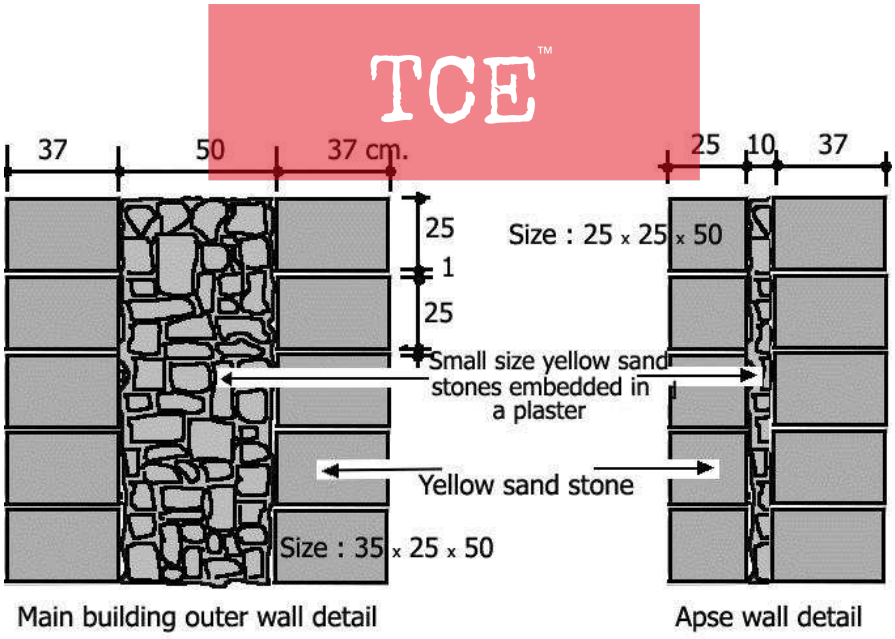

Figure 3 : Main building wall details

Cannons

Total of four cannons located on the rock to the south east of the harbour.
Distance from the Cathedral : 1062 m.
Height from the sea level : 2.10 m.
Type : Siege
Bore : 15.60 cm
Calibre : 49 cm.
Groove : No grooves
Length : 200 cm.
Max distance/impact angle : 45^0 [5]

Weight : 3,178 kg (7,000 lb)
Gunpowder : Mixture of varying amounts of sulfur (11.85%), Salt Peter – potassium nitrate- (74.64%), and charcoal (13.51%). [6]

Cannonballs

Weight : 17100 gr. (38 lb.)
Diameter : 15.50 cm
Perimeter : 48.69 cm
Impact Area : 188.69 cm^2
Muzzle velocity : 100 - 300 m/s [7]

Ball trajectory, velocity and angles

$y = ax^2 + bx + c$
condition 1 : for $y=0$, $x=0$ and $c=0$
condition 2 : for $y=(26.46-2.10)$ m., $x=1062$ m.
condition 3 : $dy/dx=1$ at $y=0$ and $x=0$
trajectory formula : $y=-0.000764x^2 + 0.834x$
or : $y = - 0.00092x^2 + x$ ……..*(1)*

from the equation $y = -\frac{g}{2v_0^2 Cos^2\theta}x^2 + (\tan\theta)x$ [8] …..*(2)*

muzzle velocity = 103.2 m/s
firing angle : 45^0 maximum
impact angle : 43.51^0
peak point, x-cordinate : $-b/2a = -(1/-2x0.00092) = 543.48$m.
peak point, y-cordinate : for $x=543.48$ $y=271.74$ m.
range : 1086.95 m.

Impact energy of cannonball

$E_{impact} = E_{horizontal} + E_{vertical}$
$E_{horizontal} = ½\ m\ V_{ox}^2$

where V_o = 103.2 m/s (cannonball's initial [muzzle] velocity),
V_{ox} = Horizontal component = cos 45 x V_o = 72.98 m/s
m=mass (kg) = w/g = 17.1/9.81 = 1.743 kg

$E_{horizontal}$= ½ x 1.743 x 72.98^2 = 4641.68 joule
$E_{vertical}$ = m x g x h where m=mass (kg), g=gravity (m/s^2), h=max. height (m.)
= 1.743 x 9.81 x 271.74 = 4646.44 joule
E_{impact} = 4641.68 + 4646.44 = 9288.12 joule (Newton-meter) = 946.80 kgf-m

Impact force of the cannonball

E_{impact} = ½ x F x ε x L [9]
Where F = force generated by the impact energy (kgf)
ε = strain (0.001 for sand stone)
L= thickness of the wall. (m.)

946.80 = ½ x F x 0.001 x 1.23
F = 1,539,512.19 kgf (Impact force)

Effect of a cannonball to a single sand stone

A = Impact Area : 188.69 cm^2

Compressive force exerted by cannonball : $\frac{F}{A} = \frac{1539512}{188.69} = 8158{,}95\ kg/cm^2$

Contact area of yellow sand stone with cannon ball : 50 x 25 = 1250 cm^2
Resisting area of yellow sand stone to cannon ball : 50 x 25 = 1250 cm^2
Resisting force of a single sand stone (1 cm thick): 1250 x 62.50 = 78,125 kg
Resisting force of a single sand stone (35 cm thick) : 78125 x 35 = 2,734,375 kg

Resisting force (2,734,375 kgf) 〈 Impact force (1,539,512 kgf)

This means that a single cannon ball can only penetrate to the wall but can not knock it down. To knock down a part of the wall, at least three cannon balls should hit the same exact point with same velocity and same angle of impact.

Effect of a cannonball hit to the apse vault

Impact angle : 43.51^0
Vertical component : Sin 43.51 = 0.6887 (downwards : -ve) ↓
Horizontal component : Cos 43.51 = 0.7250 (to the right : +ve) →
F (Impact force) = 1,539,512 kgf
$F_{vertical}$ = 1,539,512 x 0.6887 = 1,060,261.91 kgf ↓
$F_{horizontal}$ = 1,539,512 x 0.7250 = 1,116,146.20 kgf →

Figure 4 shows the dimensions and numberings of the main apse vault stones.

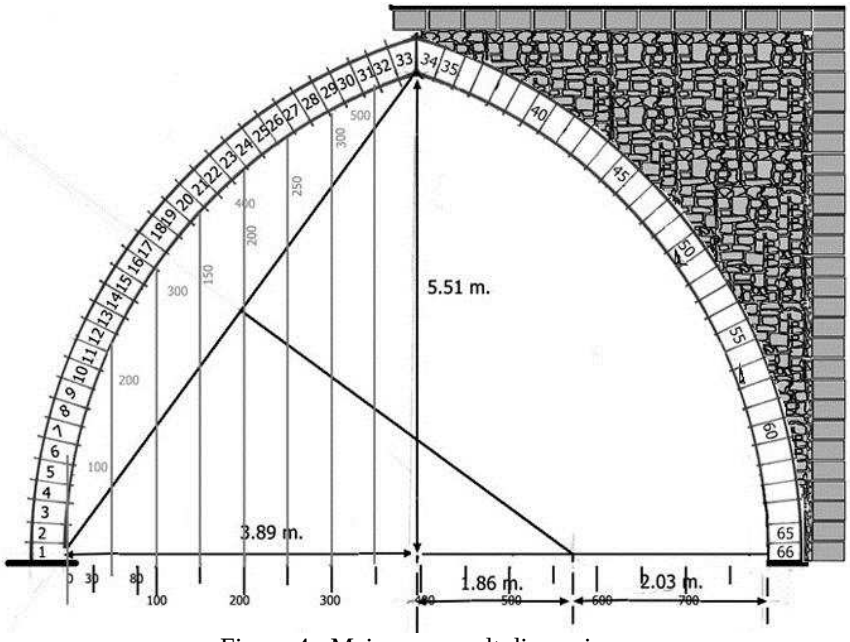

Figure 4 : Main apse vault dimensions.

Assuming direct hits to the stone no.s 27, 30 and 33, which are the most weakly covered stones of the vault.

The computer based static analysis results

Resisting Moment of yellow sand stone : $Rbd^2/6$
Where R = Compressive strength : 62.50 kg/cm^2
b = Breadth of stone = 35 cm.
d = Depth of stone = 25 cm.
M_R = 62.50 x 35 x $25^2/6$ = 227,864.58 kgf-cm
= 2,278.64 kgf-m

Hit on stone no. 27
Maximum end force occurs on stone 46 : 1,860,774 kgf
Maximum moment on stone 46 : -ve 105,182 kgf-m
End moments on stone 46 : 174,727 kgf-m
Maximum Moment 〈 Resisting Moment
No failure

Hit on stone no. 30
Maximum end force occurs on stone 34 : 3,664,973 kgf
Maximum moment on stone 34 : -ve 114,522 kgf-m
End moments on stone 34 : 232,820 kgf-m

Maximum Moment 〈 Resisting Moment
No failure

Hit on stone no. 33
Maximum end force occurs on stone 38 : 3,495,401 kgf
Maximum moment on stone 38 : +ve 483,969 kgf-m
End moments on stone 38 : 389,880 kgf-m
Maximum Moment 〉 Resisting Moment
Failure of stone 33

Conclusion

It can be seen from the results that;

a) When the cannonball hit the main side walls, it could not knock down the wall fully or partially but damage it locally, penetrating inside the outer wall 20-30 cm. with an angle of 43.51^0.
b) When the cannonball hit the main apse vault stones, No.1 to 32 and No. 35 to 66, it could not knock down the stone wholly or partially but damage the upper cover stones locally, penetrating inside the cover 20-20 cm. with an angle of 43.51^0.
c) When the cannonball hit the main apse vault stones, No.33 and 34 which are the keystones of the vault (arch) , it damaged the upper cover stones locally where the thickness was around 10 cm. and knocked down or moved the keystones, which lead to the partial fall down of the roof. The fall down ended where the side covers of the vault reached to the thickness in excess of 10 + 10 cms.

My findings lead me to the fact that the battering of the Ottoman cannonballs managed to knock down the central part of the roof around the keystone and the repair of the roof seemed very hard or impossible or was not of importance. The cannonballs hitting the side walls managed to dig holes of 20-35 cm deep only but could not severely damage or knock down the walls. Due to the scare look of the partially damaged vaulted roof, no body dared to stand under it and the building was abandoned. [1]

The earth quake which shook the whole island on 1556 [10], knock down the partially damaged and hardly standing roof completely. The earth quakes which took place on 1735 [11] [12] and 1741 [13] [14] knocked down the 80% of the already shaken walls, where most of the stones (I believe) were loose.

During the construction years of the Suez Canal which begun on 1859 [15] and city of Port Said (named after Said Pasha), the stones of the medieval buildings knocked down by the earthquakes allover in the island of Cyprus were dispatched to the area for construction purposes. This destruction of the antiquity lasted till 1905, completion of the port of Famagusta. [16]

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